

Chiral-particle Approach to Hadrons in an Extended Chiral (σ, π, ω) Mean-Field Model

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Abstract

The chiral nonlinear (σ, π, ω) mean-field model is an extension of the conserving nonlinear (nonchiral) σ - ω hadronic mean-field model which is thermodynamically consistent, relativistic and Lorentz-covariant mean-field theory of hadrons. In the extended chiral (σ, π, ω) mean-field model, all the masses of hadrons are produced by chiral symmetry breaking mechanism, which is different from other conventional chiral partner models. By comparing both nonchiral and chiral mean-field approximations, the effects of chiral symmetry breaking to the mass of σ -meson, coefficients of nonlinear interactions, coupling ratios of hyperons to nucleons and Fermi-liquid properties are investigated in nuclear matter, hyperonic matter, and neutron stars.

PACS numbers: 21.65.+f, 24.10.Cn, 24.10.Jv, 26.60.+C

1 Introduction

A renormalizable quantum field theory based on hadronic degrees of freedom provides us with an intuitively and physically accessible approach from finite nuclei to infinite nuclear matter, *e.g.*, high density nuclear and hyperonic matter such as neutron (hadronic) stars [1] – [7]. The linear neutral scalar and vector (σ, ω) , nonlinear (σ, ω) , nonlinear (σ, ω, ρ) mean-field models are actively studied and applied to finite and infinite hadronic many-body systems. Though hadronic mean-field models render nuclear and astronomical phenomena readily understandable, they are strongly interacting particles, which makes hadronic approach much complicated. One may investigate the hadronic system by starting from quantum chromodynamics (QCD), but because of strong interactions, it is complicated in the nuclear energy domain so that one is led to introduce certain effective hadronic models to simulate strong interactions of hadrons [8].

The hadronic mean-field models must be constructed to reproduce binding energy at saturation of symmetric nuclear matter (assumed as -15.75 MeV at $\rho_0 = 0.148 \text{ fm}^{-3}$ or $k_F = 1.30 \text{ fm}^{-1}$ in the current calculation), which is one of fundamental requirements for nuclear physics. The pressure must vanish at saturation ($p = 0$), and simultaneously, the self-consistent single particle energy, $E(k_F)$, must be obtained by the functional derivative of energy density with respect to baryon density, $\delta\mathcal{E}/\delta\rho_B = E(k_F)$, as a dynamical constraint for any employed approximation. The energy density and pressure must maintain a thermodynamic relation, such as $\mathcal{E} + p = \mu\rho_B$ (at $T = 0$), to be a self-consistent approximation for nuclear matter. In terms of dynamical quantities, the self-consistent requirement can be stated that Green function, self-energy and energy density must maintain conditions of conserving approximations, termed as *thermodynamic consistency*. Thermodynamic con-

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sistency is explicitly expressed as the requirement that functional derivatives of energy density with respect to self-energies must vanish, $\delta\mathcal{E}/\delta\Sigma = 0$ [9], which becomes equivalent to Landau's hypothesis of quasiparticles and the fundamental requirement of density functional theory [10] – [12].

Although the linear and nonlinear (σ, ω, ρ) mean-field models appropriately simulate properties of symmetric nuclear matter and neutron stars, they have many free parameters, masses and nonlinear coupling constants, coming from meson fields and nonlinear interactions. The upper bounds of values of nonlinear coefficients are confined by maintaining conditions of thermodynamic consistency to an employed approximation [5] and reproducing empirical data. Nonlinear coefficients are bounded for thermodynamically consistent approximations, and this is discussed as a manifestation of *naturalness* for self-consistent approximations. However, it is beyond the linear and nonlinear mean-field models to answer the reason why coupling constants for nonlinear interactions are needed and restricted with such strengths. The chiral mean-field model reveals important meanings and strength of nonlinear interactions [13] – [16]. The current chiral (σ, π, ω) mean-field approximation provides with the followings:

1. Generations of hadron masses by way of chiral symmetry breaking correspondingly produce coefficients of nonlinear meson interactions. It indicates that the fundamental requirement at nuclear matter saturation is directly related with experimental values of hadron masses $(M_N, m_\pi, m_\omega, m_\rho, \dots)$. In mean-field (Hartree) approximation, pion contributions vanish, and σ -meson compensates for attractive contributions expected to be given by pions at saturation density. Hence, the saturation property determines the mass of sigma meson, m_σ .

2. The coupling constants for hyperons are important to study phase transitions from β -equilibrium (n, p, e) asymmetric nuclear matter to (n, p, H_1, e) hyperonic matter, binding energy of pure-hyperon matter and masses of hadronic stars. It is found that Λ -hyperon coupling ratio to nucleon, $r_{\Lambda N}^\omega = g_{\omega\Lambda}/g_{\omega N}$, is expected to be $r_{\Lambda N}^\omega \sim 1.0$ by the requirement of thermodynamic consistency [6, 12], whereas the SU(6) quark model for hadrons demands $r_{\Lambda N}^\omega \sim 2/3$, or $1/3$ [18, 19]. The differences of $r_{\Lambda N}^\omega$ result in significant discrepancies in effective masses of hadrons, onset densities of nucleon-hyperon phase transitions, saturation properties of hyperons, and masses of hadron stars [6]. If the current chiral (σ, π, ω) mean-field model is applied to phase transition to β -equilibrium Lambda matter (n, p, Λ, e) , it deduces that $r_{\Lambda N}^\sigma = g_{\sigma\Lambda}/g_{\sigma N} = M_\Lambda/M_N \approx 1.187$ (see, sec. 3), which is consistent with the analysis of the conserving, nonchiral (σ, ω, ρ) mean-field approximation.

The current extended chiral (σ, π, ω) mean-field model starts from a lagrangian without hadron masses and generates all the hadron masses by way of chiral symmetry breaking. This is different from other chiral mean-field models which introduce the isoscalar-vector particle, ω , externally in order to produce repulsive interaction and saturation mechanism. The current chiral (σ, π, ω) mean-field model produces masses of σ , π and ω particles by chiral symmetry breaking mechanism. The chiral symmetric interaction and chiral breaking, binding energy are discussed in sec. 2 and Fermi-liquid properties of nuclear matter, such as incompressibility and symmetry energy, K and a_4 , and numerical results are shown in sec. 3.

The vacuum fluctuation corrections to the chiral (σ, π, ω) mean-field approximation, applications to β -equilibrium (n, p, e) asymmetric nuclear matter and properties of hadron (neutron) stars are discussed in sec. 4. The phase transition from symmetric nuclear matter to β -equilibrium hyperon matter, (n, p, H_1, e) , and important results on coupling ratios given by chiral symmetry breaking are also discussed, and concluding remarks are in sec. 5.

2 An extended chiral σ, π, ω nonlinear mean-field approximation

The conventional chiral mean-field models for hadrons suppose that the lagrangian with interaction potential, $V(\sigma^2 + \pi^2)$, should be invariant under the chiral transformation and constrain only σ and π mesons as a *chiral partner*. Moreover, a massive isoscalar vector field ω_μ is input externally to supply repulsive nuclear-nuclear interactions as in QHD-I[1, 2]. The conventional chiral mean-field models

for hadrons exhibit that when chiral symmetry breaking parameter vanishes, the masses m_σ and m_π vanish: $m_\sigma \rightarrow 0$, $m_\pi \rightarrow 0$, whereas $m_\omega \not\rightarrow 0$.

We introduce an extended chiral symmetric mean-field lagrangian for hadrons with the interaction potential, $V(\sigma^2 + \pi^2 - a\omega_\mu^2)$. The lagrangian is invariant under the chiral transformation and produces all hadron masses and nonlinear mean-field interactions by way of chiral symmetry breaking. The parameter a is constant, which will be identified as $m_\omega^2/m_\pi^2 \sim 31.65$ at nuclear domain, after the chiral symmetry breaking. Therefore, the current extended chiral mean-field model generates ω -meson as *chiral particles* such that all the meson masses are required to vanish simultaneously: $m_\sigma \rightarrow 0$, $m_\pi \rightarrow 0$, and $m_\omega \rightarrow 0$ when the chiral breaking parameter vanishes, $\varepsilon \rightarrow 0$. In other words, we assume that all the hadron masses ($M_N, m_\sigma, m_\pi, m_\omega$) and nonlinear interactions be generated by the lagrangian with interaction potential $V(\sigma^2 + \pi^2 - a\omega_\mu^2)$ under the chiral symmetry breaking mechanism.

The current extended chiral mean-field model that produces all the hadron masses ($M_N, m_\sigma, m_\pi, m_\omega$) with the chiral symmetry breaking is based on a relativistic chiral (σ, π, ω) model discussed by Walecka, Serot and others[13] – [16]. The extended chiral (σ, π, ω) lagrangian is

$$\begin{aligned} \mathcal{L} = & \bar{\psi} [\gamma_\mu (i\partial^\mu - g_\omega \omega^\mu) + g(\sigma + i\gamma_5 \boldsymbol{\tau} \cdot \boldsymbol{\pi})] \psi \\ & + \frac{1}{2} (\partial_\mu \sigma \partial^\mu \sigma + \partial_\mu \boldsymbol{\pi} \cdot \partial^\mu \boldsymbol{\pi}) - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - V(\sigma^2 + \pi^2 - a\omega_\mu^2) - \delta\mathcal{L}_{csb}, \end{aligned} \quad (2.1)$$

where $\delta\mathcal{L}_{csb} = \varepsilon\sigma$ is the chiral symmetry breaking term. The nucleon is $\psi = \begin{pmatrix} \psi_p \\ \psi_n \end{pmatrix}$, and σ, π, ω_μ are neutral scalar meson, isovector pion and neutral isovector omega meson fields, respectively, and $F_{\mu\nu} = \partial_\mu \omega_\nu - \partial_\nu \omega_\mu$ is for the vector-isoscalar ω -meson. Note that there are no baryon and meson masses in the lagrangian (2.1), and baryons and mesons are coupled as $g_\omega \bar{\psi} \gamma_\mu \omega^\mu \psi$, and $g \bar{\psi} (\sigma + i\gamma_5 \boldsymbol{\tau} \cdot \boldsymbol{\pi}) \psi$. The coupling constant, g , is pion-nucleon (and σ -nucleon) coupling constant to be required from invariance under the chiral transformation ($g_\sigma = g_\pi = g$ is assumed). We introduce the chiral-invariant potential of the form:

$$V(\sigma^2 + \pi^2 - a\omega_\mu^2) = \frac{\lambda}{4} [\sigma^2 + \pi^2 - a\omega_\mu^2]^2, \quad (2.2)$$

where $\lambda \neq 0$ and $a > 0$ are constants determined in the ground state after the chiral-symmetry breaking. Hence, the free parameters of the current chiral mean-field model are g , g_ω and λ . Note that (σ, π, ω) mesons make the lagrangian chiral invariant all together, not by way of the chiral-invariance generated by (σ, π) -chiral partner, and in this sense, we call (σ, π, ω) mesons as chiral particles.

The current chiral lagrangian is invariant under the following gauge transformations

$$\begin{aligned} \delta\psi &= \frac{i}{2} \boldsymbol{\epsilon} \cdot \boldsymbol{\tau} \gamma_5 \psi, \\ \delta\boldsymbol{\pi} &= -\boldsymbol{\epsilon} \sigma, \\ \delta\sigma &= \boldsymbol{\epsilon} \cdot \boldsymbol{\pi}, \end{aligned} \quad (2.3)$$

and $\boldsymbol{\epsilon}$ is supposed to be an infinitesimal value, and the ω meson is invariant under the gauge transformation: $\delta\omega_\mu = 0$. After the chiral symmetry breaking, the interaction potential is given in the new ground state as,

$$\begin{aligned} V &= \frac{\lambda}{4} [(\sigma^2 + \pi^2 - a\omega_\mu^2) - v^2]^2 + \delta\mathcal{L}_{csb} \\ &= \frac{\lambda}{4} [(\sigma^2 + \pi^2 - a\omega_\mu^2) - v^2]^2 + \varepsilon\sigma, \end{aligned} \quad (2.4)$$

where λ , v , a and ε are constants determined at the ground state.

The mesons are excited from the new ground state as follows,

$$\begin{aligned}\sigma &\rightarrow \langle \sigma \rangle + \phi, \\ \boldsymbol{\pi} &\rightarrow \langle \boldsymbol{\pi} \rangle + \boldsymbol{\pi}, \\ \omega_\mu &\rightarrow \langle \omega_\mu \rangle + \omega_\mu,\end{aligned}\tag{2.5}$$

where $\langle \sigma \rangle$, $\langle \boldsymbol{\pi} \rangle$ and $\langle \omega_\mu \rangle$ are values for the meson fields in the vacuum defined by minimization of (2.4) with respect to σ , $\boldsymbol{\pi}$, and ω_μ . The interaction potential V has the following form at the ground state in the new vacuum,

$$V = \frac{\lambda}{4} \left[\left(\langle \sigma \rangle^2 + \langle \boldsymbol{\pi} \rangle^2 - a \langle \omega_\mu \rangle^2 \right) - v^2 \right]^2 + \varepsilon \langle \sigma \rangle, \tag{2.6}$$

and the minimization conditions give

$$\begin{aligned}\frac{\partial V}{\partial \langle \sigma \rangle} &= \lambda \langle \sigma \rangle \left[\left(\langle \sigma \rangle^2 + \langle \boldsymbol{\pi} \rangle^2 - a \langle \omega_\mu \rangle^2 \right) - v^2 \right] + \varepsilon = 0, \\ \frac{\partial V}{\partial \langle \boldsymbol{\pi} \rangle} &= \lambda \langle \boldsymbol{\pi} \rangle \left[\left(\langle \sigma \rangle^2 + \langle \boldsymbol{\pi} \rangle^2 - a \langle \omega_\mu \rangle^2 \right) - v^2 \right] = 0, \\ \frac{\partial V}{\partial \langle \omega_\mu \rangle} &= \lambda a \langle \omega_\mu \rangle \left[\left(\langle \sigma \rangle^2 + \langle \boldsymbol{\pi} \rangle^2 - a \langle \omega_\mu \rangle^2 \right) - v^2 \right] = 0.\end{aligned}\tag{2.7}$$

The conditions, $\lambda \neq 0$ and $\varepsilon \neq 0$, lead to $\langle \sigma \rangle \equiv \sigma_0 \neq 0$, $[\langle \sigma \rangle^2 + \langle \boldsymbol{\pi} \rangle^2 - a \langle \omega_\mu \rangle^2 - v^2] \neq 0$, and

$$\langle \boldsymbol{\pi} \rangle = 0, \quad \langle \omega_\mu \rangle = 0. \tag{2.8}$$

The ground state value, σ_0 , is then defined as,

$$\langle \sigma \rangle \equiv \sigma_0 = -\frac{M}{g}. \tag{2.9}$$

By expanding the interaction potential $V(\sigma^2 + \boldsymbol{\pi}^2 - a\omega_\mu^2)$, the terms in (2.6) are collected as follows:

(1) Constant terms are

$$V_0 = \frac{\lambda}{4}(\sigma_0^2 - v^2)^2 + \varepsilon \sigma_0. \tag{2.10}$$

(2) The terms linear in ϕ are

$$V_1 = \{ \lambda \sigma_0 (\sigma_0^2 - v^2) + \varepsilon \} \phi = 0. \tag{2.11}$$

This expression vanishes because of the minimization conditions, (2.7) and (2.8).

(3) The terms quadratic in $\boldsymbol{\pi}$ are

$$V_2 = -\frac{1}{2} \frac{\varepsilon}{\sigma_0} \boldsymbol{\pi}^2 = \frac{1}{2} \frac{g\varepsilon}{M} \boldsymbol{\pi}^2 \equiv \frac{1}{2} \mu_2^2 \boldsymbol{\pi}^2. \tag{2.12}$$

(4) The terms quadratic in ω_μ are derived in the same way as,

$$V_3 = -\frac{1}{2} \frac{g\varepsilon}{M} a \omega_\mu^2 \equiv -\frac{1}{2} \mu_3^2 \omega_\mu^2. \tag{2.13}$$

(5) The terms quadratic in ϕ are

$$V_4 = \frac{\lambda}{2} (\sigma_0^2 - v^2) \phi^2 + \lambda \sigma_0^2 \phi^2 \equiv \frac{1}{2} \mu_1^2 \phi^2. \tag{2.14}$$

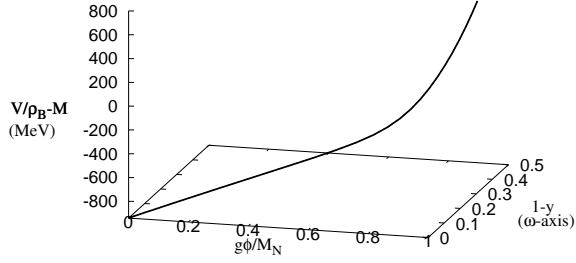


Fig.1. The interaction potential V defined by meson sectors. The field ϕ produces attractive interaction at low densities. The ω -axis is written by the variable $1 - y$, where $y = (m_\omega^2/g_\omega\rho_B)\omega_0$, and ω -field produces repulsive interaction at high densities.

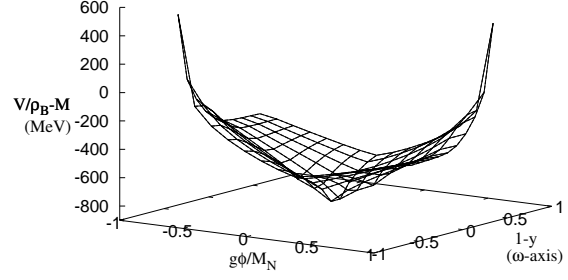


Fig.2. The 3-dimensional image of interaction potential V . The origin is set in the center of ϕ - ω plain.

and λ is given by

$$\lambda \equiv \frac{1}{2} \left(\frac{g}{M} \right)^2 (\mu_1^2 - \mu_2^2). \quad (2.15)$$

(6) The remaining cubic and quartic interactions of the meson fields (ϕ, π, ω) are then given by

$$\begin{aligned} V_5 + V_6 + V_7 &= \frac{\lambda}{4} [4\sigma_0\phi(\phi^2 + \pi^2 - a\omega_\mu^2) + (\phi^2 + \pi^2 - a\omega_\mu^2)^2] \\ &= \frac{1}{2}(\mu_1^2 - \mu_2^2) \left[\left(\frac{g}{2M} \right)^2 (\phi^2 + \pi^2 - a\omega_\mu^2)^2 - 2 \left(\frac{g}{2M} \right) \phi(\phi^2 + \pi^2 - a\omega_\mu^2) \right]. \end{aligned} \quad (2.16)$$

A collection of these terms then yields the interaction potential V written as,

$$\begin{aligned} V &= \frac{1}{2}\mu_1^2\phi^2 + \frac{1}{2}\mu_2^2\pi^2 - \frac{1}{2}\mu_3^2\omega_\mu^2 \\ &+ \frac{1}{2}(\mu_1^2 - \mu_2^2) \left[\left(\frac{g}{2M} \right)^2 (\phi^2 + \pi^2 - a\omega_\mu^2)^2 - 2 \left(\frac{g}{2M} \right) \phi(\phi^2 + \pi^2 - a\omega_\mu^2) \right]. \end{aligned} \quad (2.17)$$

The lagrangian density (2.1) with the generation of hadron masses by spontaneous symmetry breaking finally takes the following form:

$$\begin{aligned} \mathcal{L}_{csb} &= \bar{\psi} [\gamma_\mu (i\partial^\mu - g_\omega\omega^\mu) - \{M - g(\phi + i\gamma_5\boldsymbol{\tau} \cdot \boldsymbol{\pi})\}] \psi \\ &+ \frac{1}{2}(\partial_\mu\phi\partial^\mu\phi - \mu_1^2\phi^2) + \frac{1}{2}(\partial_\mu\boldsymbol{\pi} \cdot \partial^\mu\boldsymbol{\pi} - \mu_2^2\pi^2) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}\mu_3^2\omega_\mu^2 \\ &- \frac{1}{2}(\mu_1^2 - \mu_2^2) \left[\left(\frac{g}{2M} \right)^2 (\phi^2 + \pi^2 - a\omega_\mu^2)^2 - 2 \left(\frac{g}{2M} \right) \phi(\phi^2 + \pi^2 - a\omega_\mu^2) \right] + \text{constant}. \end{aligned} \quad (2.18)$$

The parameters are identified to be: $\mu_1 = m_\sigma, \mu_2 = m_\pi, \mu_3 = m_\omega$, and $a \equiv m_\omega^2/m_\pi^2 \sim 31.65$ in nuclear domain.

The chiral $(\sigma, \boldsymbol{\pi}, \omega)$ mean-field approximation is defined by replacing meson quantum fields with classical fields: $\hat{\phi} \rightarrow \phi_0, \hat{\omega}_\mu = (\hat{\omega}_0, \boldsymbol{\omega}) \rightarrow (\omega_0, \mathbf{0})$, they are constants independent of x_μ . The spacial

part of the vector field $\langle \omega \rangle$ should vanish by the requirement of rotational invariance of static and homogeneous nuclear matter [1], and in addition, π -meson contributions vanish in the (mean-field) Hartree approximation. The chiral mean-field lagrangian is given by

$$\mathcal{L}_{csb} = \bar{\psi} [\gamma_\mu (i\partial^\mu - g_\omega \omega_0) - (M - g\phi_0)] \psi - \frac{1}{2} m_\sigma^2 \phi_0^2 + \frac{1}{2} m_\omega^2 \omega_0^2 - \frac{1}{2} (m_\sigma^2 - m_\pi^2) \left[\left(\frac{g}{2M} \right)^2 (\phi_0^2 - a\omega_0^2)^2 - 2 \left(\frac{g}{2M} \right) \phi_0 (\phi_0^2 - a\omega_0^2) \right]. \quad (2.19)$$

The equations of motion for the scalar and vector mesons are given by

$$m_\sigma^2 \phi_0^2 - \frac{g}{2M} (m_\sigma^2 - m_\pi^2) (3\phi_0^2 + 2a\phi_0 \omega_0^2 - 2\frac{g}{2M} \phi_0^3) = g\rho_s^*, \quad (2.20)$$

$$m_\omega^2 \omega_0 - 2\frac{g}{2M} (m_\sigma^2 - m_\pi^2) \left(a\phi_0 \omega_0 + \frac{g}{2M} a^2 \omega_0^3 - \frac{g}{2M} a\phi_0^2 \omega_0 \right) = g_\omega \rho_B, \quad (2.21)$$

where ρ_s^* is the scalar source, and ρ_B is the baryon density: $\rho_B = \sum_B k_{FB}^3 / 3\pi^2$, where k_{FB} is a baryon Fermi-momentum. The energy density and pressure can be derived from energy momentum tensor [2, 5]:

$$\mathcal{E} = \sum_{B=n,p} \frac{2}{(2\pi)^3} \int^{k_{FB}} d^3k E_B(k) + \frac{1}{2} m_\sigma^2 \phi_0^2 - \frac{g}{2M} (m_\sigma^2 - m_\pi^2) \left(\phi_0^3 - \frac{1}{2} \frac{g}{2M} \phi_0^4 \right) - \frac{1}{2} m_\omega^2 \omega_0^2 + \frac{g}{2M} (m_\sigma^2 - m_\pi^2) a \left(\phi_0 \omega_0^2 + \frac{1}{2} \frac{g}{2M} a \omega_0^4 - \frac{g}{2M} \phi_0^2 \omega_0^2 \right), \quad (2.22)$$

$$p = \sum_{B=n,p} \frac{1}{3} \frac{2}{(2\pi)^3} \int^{k_{FB}} d^3k \frac{k^2}{E_B^*(k)} - \frac{1}{2} m_\sigma^2 \phi_0^2 + \frac{g}{2M} (m_\sigma^2 - m_\pi^2) \left(\phi_0^3 - \frac{1}{2} \frac{g}{2M} \phi_0^4 \right) + \frac{1}{2} m_\omega^2 \omega_0^2 - \frac{g}{2M} (m_\sigma^2 - m_\pi^2) a \left(\phi_0 \omega_0^2 + \frac{1}{2} \frac{g}{2M} a \omega_0^4 - \frac{g}{2M} \phi_0^2 \omega_0^2 \right), \quad (2.23)$$

where $E_B(k) = E_B^*(k) + \Sigma_\omega^0 = \sqrt{k^2 + M_B^{*2}} - g_\omega \omega_0$. The scalar source ρ_s^* is derived from the functional derivative with respect to ϕ_0 [5]:

$$\rho_s^* = \sum_B \frac{1}{\pi^2} \int^{k_{FB}} q^2 dq \frac{M^*}{E^*(q)} - \frac{1}{2M} (m_\sigma^2 - m_\pi^2) a \omega_0^2. \quad (2.24)$$

The self-consistent effective masses of hadrons are determined by satisfying conditions of thermodynamic consistency [5]:

$$\begin{aligned} M_N^* &= M - g\phi_0, \\ m_\sigma^{*2} &= m_\sigma^2 - \frac{3g}{2M} (m_\sigma^2 - m_\pi^2) \phi_0 + 2(m_\sigma^2 - m_\pi^2) \left(\frac{g}{2M} \right)^2 (\phi_0^2 - a\omega_0^2), \\ m_\omega^{*2} &= m_\omega^2 - \frac{g}{M} a (m_\sigma^2 - m_\pi^2) \phi_0 + 2a(m_\sigma^2 - m_\pi^2) \left(\frac{g}{2M} \right)^2 - 2a^2 (m_\sigma^2 - m_\pi^2) \left(\frac{g}{2M} \right)^2 \omega_0^2, \end{aligned} \quad (2.25)$$

and self-consistent scalar and vector self-energies are given by [5]:

$$\Sigma^s = -\frac{g^2}{m_\sigma^{*2}} \rho_s^*, \quad \Sigma_\omega^\mu = -\frac{g_\omega^2}{m_\omega^{*2}} \rho_B \delta_{\mu,0}, \quad (2.26)$$

where m_σ^* and m_ω^* are effective masses of σ and ω mesons.

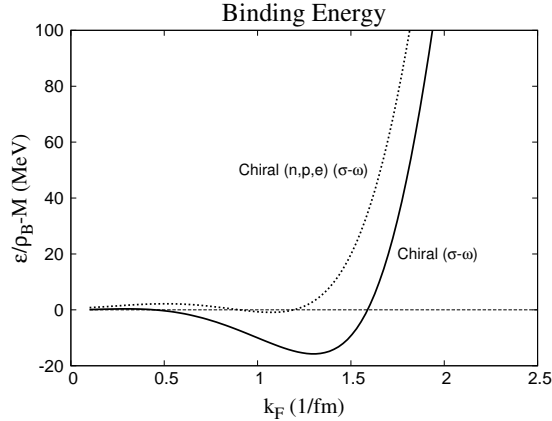


Fig.3. The binding energies of isospin symmetric (n,p) and isospin asymmetric (n,p,e) matter. Note that $\mathcal{E}/\rho_B = E(k_F)$ is exactly satisfied at saturation density, $\rho_B = 0.148 \text{ fm}^{-3}$.

The interaction potential defined by meson sectors is shown in Fig. 1 and a three-dimensional image of the interaction potential is shown in Fig. 2. In the current chiral mean-field approximation, the interaction potential is self-consistently constructed by σ and ω mesons; σ -meson produces attractive interaction at low densities, whereas ω -meson mainly generates repulsive contributions at high densities. The energy density and pressure satisfy, $\mathcal{E} + p = \mu\rho_B$ and $\mu = E(k_F)$ in all densities. The binding energies of symmetric nuclear matter and (n,p,e) asymmetric matter are shown in Fig. 3.

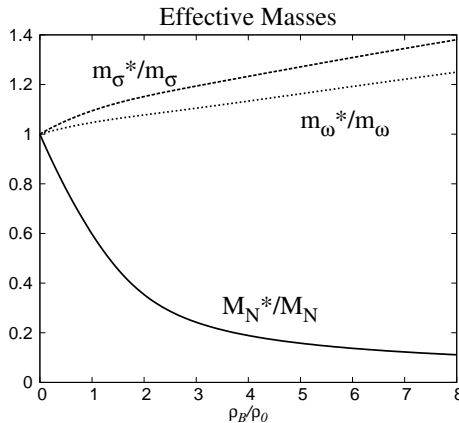


Fig.4. Effective masses of nucleon, M_N^*/M_N , and mesons, m_σ^*/m_σ and m_ω^*/m_ω . The qualitative behavior of effective masses are consistent with those derived from nonlinear, nonchiral (σ, ω, ρ) mean-field approximation.

3 Fermi liquid properties at nuclear matter saturation

The chiral (σ, π, ω) mean-field model exhibits remarkable properties when it is compared to the nonchiral, nonlinear (σ, ω, ρ) mean-field model. The nonchiral mean-field model is applied to (n,p) symmetric, (n,p,e) asymmetric, (n,p,H,e) hyperonic matter, and neutron stars [6]. Although the nonchiral model reasonably simulates properties of nuclear and neutron matter, it has many parameters such as masses and coupling constants. The upper and lower bound values of coupling constants and effective masses of hadrons are constrained by empirical data and self-consistent conditions to approximations. The nonlinear nonchiral mean-field approximations could not clearly explain the reason why values

of nonlinear coupling constants are bound in a characteristic way [6]. The chiral (σ, π, ω) mean-field model clarifies relations among nonlinear coupling constants, hadron masses and observables.

All the hadron masses and nonlinear coefficients are related to properties of symmetric nuclear matter, such as binding energy, K and a_4 , because the chiral breaking mechanism determines nonlinear interactions in terms of hadron masses and coupling constants, g and g_ω . Consequently, the mass of σ -meson, m_σ , is related to the binding energy of symmetric nuclear matter ($\mathcal{E}/\rho_B - M = -15.75$ MeV, at $k_F = 1.30$ fm $^{-1}$) and adjusted self-consistently. The incompressibility is calculated by

$$K = 9\rho_B \frac{\partial^2 \mathcal{E}}{\partial \rho_B^2} = 9\rho_B \left(\frac{\partial \mu}{\partial \rho_B} \right), \quad (3.1)$$

where μ is the chemical potential and equal to the Fermi energy, $\mu = E(k_F)$, because the current chiral mean-field approximation is thermodynamically consistent and Landau's hypothesis for quasiparticles is maintained exactly. The symmetry energy is calculated by

$$a_4 = \frac{1}{2} \rho_B \left[\left[\frac{\partial^2 \mathcal{E}}{\partial \rho_3^2} \right]_{\rho_B} \right]_{\rho_3=0}, \quad (3.2)$$

where ρ_3 is the difference between the proton and neutron density: $\rho_3 = \rho_p - \rho_n = (k_{F_p}^3 - k_{F_n}^3)/3\pi^2$ at a fixed baryon density, $\rho_B = \rho_p + \rho_n = 2k_F^3/3\pi^2$.

The coupling constants and effective masses of hadrons, Fermi-liquid properties of symmetric nuclear matter are listed in the table 1. The effective masses of mesons are shown in Fig. 4: $M_N^*/M_N \sim 0.60$, $m_\sigma^*/m_\sigma \sim 1.09$, $m_\omega^*/m_\omega \sim 1.04$, at saturation density. The effective mass of nucleon, $M_N^*/M_N \sim 0.60$, would be considered to produce a hard EOS and large masses of neutron stars in nonchiral mean-field approximations, but the chiral mean-field approximation produces a softer EOS.

The incompressibility and symmetry energy are shown in Fig. 5 and Fig. 6, respectively, and they are $K = 371$ MeV and $a_4 = 17.4$ MeV, at saturation density. These observables are expected to be, $K \sim 300$ MeV and $a_4 \sim 30$ MeV, in the nonchiral, nonlinear (σ, ω, ρ) mean-field approximation [6]. One can notice that ρ -meson contribution would be important when a_4 in the nonchiral (σ, ω, ρ) is compared to that of chiral (σ, ω) in Fig. 6. Hence, in order to examine calculations quantitatively, the chiral (σ, π, ω) model must be extended to the chiral $(\sigma, \pi, \omega, \rho)$ model [17], which is expected to clarify chiral hadronic models.

The mass of σ meson is important because all the other observables, EOS, and masses of neutron stars, depend only on the three adjustable parameters: m_σ and coupling constants, g and g_ω . Therefore, the binding energy of symmetric nuclear matter ($\mathcal{E}/\rho_B - M = -15.75$ MeV, at $k_F = 1.30$ fm $^{-1}$) and the maximum mass of neutron stars ($M_{\max} \simeq 2.50$ M $_\odot$) determine the mass of σ meson to be

Table 1. Coupling constants and Fermi-liquid properties of nuclear matter

g	g_ω	m_σ		
2.4095	13.4232	120.0		
M_N^*/M_N	m_σ^*/m_σ	m_ω^*/m_ω	K (MeV)	a_4 (MeV)
0.60	1.09	1.04	371	17.4
M_{\max}	\mathcal{E}_c	I	R (km)	
2.60	1.58	418	12.8	

In the current chiral mean-field approximation, the masses of π and ω mesons are identified as, $\mu_2 = m_\pi = 139.0$ MeV and $\mu_3 = m_\omega = 783.0$ MeV, in the nuclear domain. Hence, adjustable parameters are only g , g_ω , and m_σ . The effective masses, K and a_4 , are values at saturation of nuclear matter: $\rho_B = 0.148$ fm $^{-3}$ (confer, table 2.).

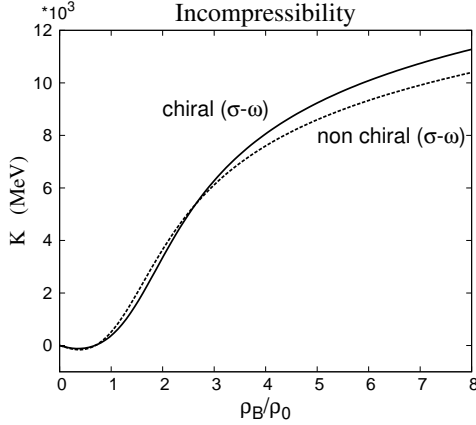


Fig.5. Incompressibilities in the nonchiral and chiral (σ, ω) mean-field approximations. The effect of chiral symmetry to incompressibilities is not significant around saturation but important at high densities.

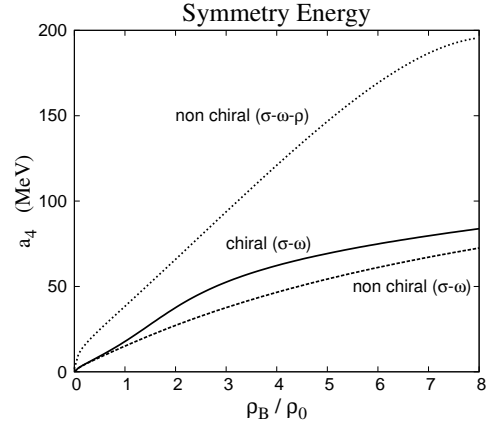


Fig.6. Symmetry energies in the nonchiral (σ, ω, ρ) isospin asymmetric matter, and nonchiral, chiral (σ, ω) isospin symmetric matter. The ρ -meson contribution is more important for a_4 .

$m_\sigma \approx 120.0$ MeV. In (Hartree) mean-field approximations, contributions of π -meson vanish in infinite matter due to spin-saturation, and hence, σ -meson compensates for π -meson contributions in order to produce the saturation mechanism of symmetric nuclear matter. The σ -meson produces attractive interactions at low densities with the mass: $m_\sigma \approx 120.0$ MeV which is close to the pion mass. Moreover, $m_\sigma \lesssim m_\pi$ is required to obtain solutions consistent with those of the conserving nonchiral mean-field approximations. If one assumes $m_\sigma > m_\pi$, solutions are restricted to low densities, but the chiral mean-field approximation is not appropriate in this case because the interaction potential V shown in Fig. 1 and 2 becomes unbound and decreases at high densities.

4 The vacuum fluctuation corrections and Neutron star properties

The full relativistic chiral Hartree approximation including vacuum fluctuation corrections (VFC) is derived in this section and applied to properties of neutron stars. The divergent integrals coming from occupied negative energy Dirac vacuum will be rendered finite by including appropriate counterterms in the current chiral lagrangian. By applying the method discussed in the linear σ - ω mean-field approximation [1] to the nonlinear σ - ω - ρ mean-field approximation [5], the baryon and meson propagators, self-energies are defined, and appropriate counterterms that render divergent integrals finite are introduced.

The baryon propagator in mean-field (Hartree) approximation is supposed to be [1]:

$$G_B^H(k) = (\gamma_\alpha k^\alpha + M_B^*) \left\{ \frac{1}{k^2 - M_B^* + i\epsilon} + \frac{i\pi}{E_B^*(k)} \delta(k^0 - E_B(k)) \theta(k_{FB} - |\mathbf{k}|) \right\} \quad (4.1)$$

$$= G_B^F(k) + G_B^D(k),$$

where $G_B^F(k)$, ($B = n, p, \Lambda, \dots$), is the propagator for negative energy Dirac-sea and $G_B^D(k)$ is for density-dependent Fermi-sea particles, respectively. It can be readily shown that energy density, pressure and self-energies in sec. 2 are computed by assuming $G^H(k) = G_B^D(k)$, and hence, we recalculate (2.26) by including $G^F(k)$, which requires renormalization of infinities into physical parameters of the model. By employing the full propagator (4.1) in chiral nonlinear σ - ω Hartree approximation, the

vector meson self-energy in eq. (2.26) becomes

$$\begin{aligned}\Sigma_\omega^\mu &= -i \frac{g_\omega}{m_\omega^{*2}} \sum_B \int \frac{d^4 k}{(2\pi)^4} \text{Tr} [i g_\omega \gamma_\mu G_B^H(k)] \\ &= 4i \frac{g_\omega}{m_\omega^{*2}} \sum_B g_\omega \int \frac{d^4 k}{(2\pi)^4} \frac{k^\mu}{k^2 - M_B^{*2} + i\epsilon} - \frac{g_\omega^2}{m_\omega^{*2}} \rho_\omega \delta_{\mu,0}.\end{aligned}\quad (4.2)$$

The first term of vector self-energy (4.2) is a divergent integral evaluated by using the technique of dimensional regularization as follows,

$$\Sigma_\omega^\mu = 4i \frac{g_\omega}{m_\omega^{*2}} \sum_B g_\omega \int \frac{d^n k}{(2\pi)^4} \frac{k^\mu}{k^2 - M_B^{*2} + i\epsilon} - \frac{g_\omega^2}{m_\omega^{*2}} \rho_\omega \delta_{\mu,0}, \quad (4.3)$$

where the first term of integration is performed in n dimensions, and the final result of any calculation will be obtained by taking the physical limit $n \rightarrow 4$. The integral (4.3) vanishes by symmetric integration, and the fact indicates that counterterm corrections (CTC) for the chiral mean-field (Hartree) approximation are produced only by way of ϕ fields.

The counterterms to make the scalar self-energy finite are evaluated by expanding the full propagator of G^H in a power series in the *renormalized* scalar self-energy Σ^σ . Using the Dyson equation, G^H is formally expanded as,

$$\begin{aligned}G^H(k) &= G^0(k) + G^0(k) \Sigma^s G^H(k) \\ &= \sum_{m=0}^{\infty} [G^0(k)]^{m+1} [\Sigma^s]^m,\end{aligned}\quad (4.4)$$

and insertion of this expression into the scalar self-energy produces,

$$\Sigma_H^s = i \frac{g_\sigma}{m_\sigma^{*2}} \sum_B \int \frac{d^n q}{(2\pi)^4} \text{Tr} \left[\sum_{m=0}^{\infty} \frac{1}{m!} [\Sigma^s]^m \frac{\partial^m G^0(q)}{\partial M^m} \right] - \frac{g_\sigma^2}{m_\sigma^{*2}} \rho_s^* + \Sigma_{\text{CTC}}^s. \quad (4.5)$$

It is clearly shown that the terms of $m = 0, 1, 2, 3$ in (4.5) have divergence when the power counting of q is performed in physical dimension $n = 4$. These divergences can be removed by including the counterterm contribution in the lagrangian density:

$$\mathcal{L}_{\text{CTC}} = \alpha_1 \phi + \frac{1}{2!} \alpha_2 \phi^2 + \frac{1}{3!} \alpha_3 \phi^3 + \frac{1}{4!} \alpha_4 \phi^4. \quad (4.6)$$

The coefficients of $\alpha_1, \alpha_2, \alpha_3, \alpha_4$ are evaluated explicitly by dimensional regularization[1]. They are given by

$$\begin{aligned}\alpha_1 &= \frac{g}{4\pi^2} \{ \Gamma(1 - n/2) + 2 \ln M_B + O(n - 4) \}, \\ \alpha_2 &= -\frac{g^2}{4\pi^2} 3M_B^2 \left\{ \Gamma(1 - n/2) + 2 \ln M_B + \frac{2}{3} + O(n - 4) \right\}, \\ \alpha_3 &= \frac{g^3}{4\pi^2} \{ 6M_B \Gamma(1 - n/2) + 12M_B \ln M + 10M_B + O(n - 4) \}, \\ \alpha_4 &= -\frac{g^4}{4\pi^2} \{ 6\Gamma(1 - n/2) + 12 \ln M_B + 22 + O(n - 4) \}.\end{aligned}\quad (4.7)$$

The lagrangian density, \mathcal{L}_{CTC} , is related to the self-energy Σ_{CTC}^s by the functional derivative as,

$$\Sigma_{\text{CTC}}^s = -\frac{g}{m_\sigma^{*2}} \frac{\delta \mathcal{L}_{\text{CTC}}}{\delta \phi_0}, \quad (4.8)$$

and the full self-energy is finally calculated as,

$$\begin{aligned} \Sigma_H^\sigma = & i \frac{g}{m_\sigma^{*2}} \sum_B \frac{g}{2\pi^2} \left[M_B^{*3} \ln \frac{M_B^*}{M_B} - M_B^2 (M_B^* - M_B) \right. \\ & \left. - \frac{5}{2} M_B (M_B^* - M_B)^2 - \frac{11}{6} (M_B^* - M_B)^3 \right] - \frac{g^2}{m_\sigma^{*2}} \rho_s^*. \end{aligned} \quad (4.9)$$

The full energy density is calculated by energy-momentum tensor and (4.6), (4.7) as,

$$\mathcal{E}_{\text{Dirac}}(M_B^*) = \frac{2\pi^{n/2}}{(2\pi)^4} \Gamma(-n/2) M_B^{*n}, \quad (4.10)$$

and the vacuum expectation value of energy density defined in the limit $k_F \rightarrow 0$ is given by,

$$\mathcal{E}_{\text{Dirac}}(M_B) = \frac{2\pi^{n/2}}{(2\pi)^4} \Gamma(-n/2) M_B^n. \quad (4.11)$$

The finite vacuum fluctuation correction to energy density is determined from (4.6) as $(-\langle\psi|\mathcal{L}_{\text{CTC}}|\psi\rangle)$, and it is calculated as:

$$\begin{aligned} \Delta\mathcal{E}_{\text{VFC}} = & \mathcal{E}_{\text{Dirac}}(M_B^*) - \mathcal{E}_{\text{Dirac}}(M_B) - \alpha_1\phi - \frac{1}{2!}\alpha_2\phi^2 - \frac{1}{3!}\alpha_3\phi^3 - \frac{1}{4!}\alpha_4\phi^4 \\ = & -\frac{1}{8\pi^2} \sum_B \left[M_B^{*4} \ln \left(\frac{M_B^*}{M_B} \right) + M_B^3 (M_B - M_B^*) - \frac{7}{2} M_B^2 (M_B - M_B^*)^2 \right. \\ & \left. + \frac{13}{3} M_B (M_B - M_B^*)^3 - \frac{25}{12} (M_B - M_B^*)^4 \right] \end{aligned} \quad (4.12)$$

and pressure is given by $\Delta p_{\text{VFC}} = -\Delta\mathcal{E}_{\text{VFC}}$, which is obtained by energy-momentum tensor as: $p = \frac{1}{3} \langle T^{ii} \rangle$, ($i = x, y, z$). The VFC gives repulsive contributions for all densities. The model parameters, m_σ , g and g_ω must be adjusted and fixed to reproduce saturation of nuclear matter, where pressure $p = 0$ and $\mathcal{E}/\rho_B = E(k_F)$ must be satisfied.

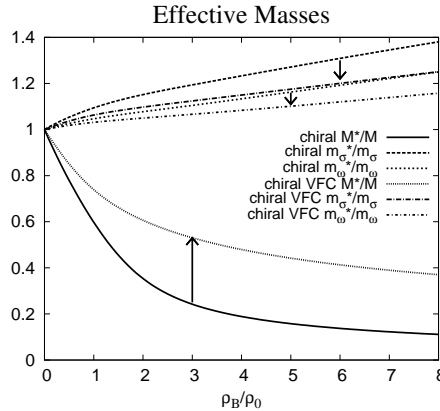


Fig.7. The vacuum fluctuation corrections to effective masses in the chiral model.

The effective masses of baryons and mesons including VFC are shown in Fig. 7, and at saturation density, they are $M_N^*/M_N \sim 0.74$, $m_\sigma^*/m_\sigma \sim 1.06$, $m_\omega^*/m_\omega \sim 1.03$; meson effective masses are almost unity around saturation. The baryon effective mass increases slightly at saturation, which produces a softer EOS at high densities and decreases the masses of neutron stars. The scalar source is decreased a little by VFC, and accordingly, other fields are similarly decreased by self-consistent relations required

by thermodynamic consistency. The coupling constants and effective masses of hadrons, Fermi-liquid properties of symmetric nuclear matter including VFC are listed in the table 2.

The incompressibility and symmetry energy with VFC are shown in Fig. 8 and Fig. 9. These Fermi-liquid properties are almost similar at saturation density, but incompressibility, K , is softened at high densities. This character shows that the effect of VFC is noticeable at high densities, but not so important at low densities. The symmetry energy including VFC gives similar results as discussed in sec. 3, and one can check from the Fig. 9 that the dominant contribution to a_4 should be expected from ρ -meson contributions. The Fock-exchange corrections produce important contributions to a_4 and K [21]. Since chiral symmetry breaking model confines coupling constants strictly, it is important to extend chiral models with ρ -meson to the conserving, chiral Hartree-Fock and Brueckner HF approximations. The phase transition from β -equilibrium (n, p, e) to (n, p, Λ, e) or (n, p, Σ^-, e)

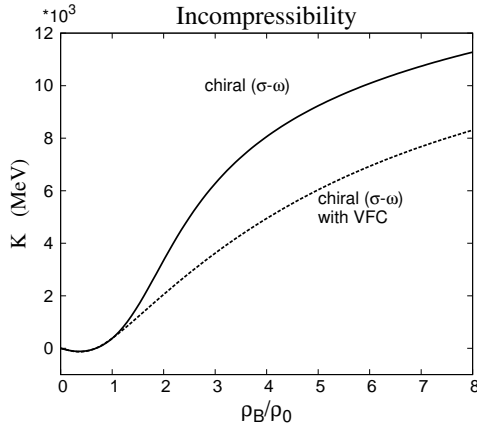


Fig.8. The vacuum fluctuation corrections to incompressibility in the chiral model.

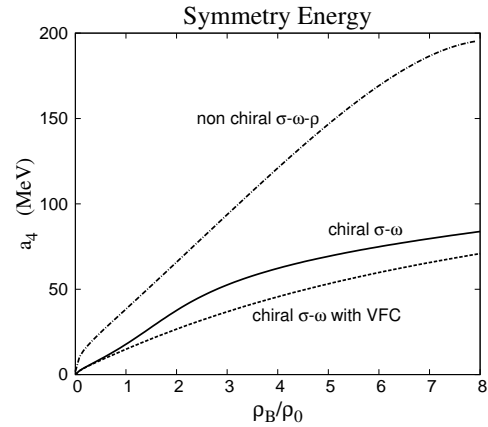


Fig.9. The vacuum fluctuation corrections to symmetry energy. The nonchiral (σ, ω, ρ) calculation is listed for comparison.

matter is discussed in the article [6]. The hyperon-onset densities depend explicitly on nucleon-hyperon coupling ratios, $r_{HN}^\omega = g_{\omega H}/g_{\omega N}$ and $r_{HN}^\sigma = g_{\sigma H}/g_{\sigma N}$ ($H = \Lambda$ or Σ^-), and they are given by

$$r_{HN}^\omega = \frac{m_\omega^{*2}}{g_{\omega N} g_{\omega N}^* \rho_\omega} \left(\frac{g_{\sigma H}}{g_{\sigma N}^*} (M_N - M_N^*) + \alpha_H \right) = \frac{m_\omega^{*2}}{g_{\omega N} g_{\omega N}^* \rho_\omega} (M_H - M_H^* + \alpha_H), \quad (4.13)$$

where $\rho_\omega = \rho_p + \rho_n$, and $g_{\omega N}^*$ is a density-dependent coupling constant; α_H is the lowest binding

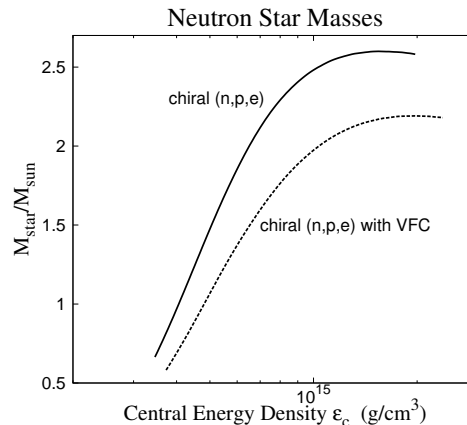


Fig.10. The masses of neutron stars in the chiral mean-field approximation, with or without VFC.

Table 2. Coupling constants and Fermi-liquid properties of nuclear matter with VFC

g	g_ω	m_σ		
1.972	10.2235	120.0		
M_N^*/M_N	m_σ^*/m_σ	m_ω^*/m_ω	K (MeV)	a_4 (MeV)
0.74	1.06	1.03	383	14.8
M_{max}	\mathcal{E}_c	I	R (km)	
2.19	1.88	249	11.6	

The result indicates that ρ -meson is necessary to obtain reasonable results for properties of Fermi-liquid and neutron stars. The analysis with chiral $(\sigma, \pi, \omega, \rho)$ model [17] is needed to extract quantitative results. M_{max} is the maximum mass in the solar mass unit (M_\odot) and $\mathcal{E}_c(10^{15}\text{g/cm}^3)$ is the central energy density; I is the inertial mass ($M_\odot\text{km}^2$) and $R(\text{km})$ is the radius of a (n, p, e) asymmetric neutron star.

energy of a hyperon. The coupling ratios are required to be $r_{\Lambda N}^\omega \sim 1.0$ and $r_{\Sigma^- N}^\omega \sim 1.0$ in the nonchiral, nonlinear (σ, ω, ρ) mean-field approximation in order to obtain optimum empirical values of symmetric nuclear matter and neutron stars. If the chiral symmetry breaking is applied to phase transitions from (n, p, e) to (n, p, Λ, e) or (n, p, Σ^-, e) matter, it supports the results that coupling ratios should be $r_{HN}^\omega \sim 1.0$, which is explained as follows. In (σ, π, ω) chiral symmetry breaking models, σ -meson generates the mass of nucleon in the new ground state: $\sigma \rightarrow \sigma_0 + \phi$ and $\sigma_0 = -M_N/g_{\sigma N}$. Let us include baryons $(n, p, \Lambda, \Sigma^-, \dots)$ into the lagrangian (2.1) and σ -hyperon coupling constants are $g_{\sigma n}, g_{\sigma p}, g_{\sigma \Lambda}, g_{\sigma \Sigma^-}, \dots$, respectively. Suppose that the ground state expectation value σ_0 is equipartitioned to baryons in the new ground state after chiral symmetry breaking. Then, one obtains $-M_n/g_n = -M_p/g_p = -M_\Lambda/g_\Lambda = -M_{\Sigma^-}/g_{\Sigma^-}$, and it results in,

$$r_{pn}^\sigma = \frac{g_{\sigma p}}{g_{\sigma n}} = \frac{M_p}{M_n}, \quad r_{\Lambda n}^\sigma = \frac{g_{\sigma \Lambda}}{g_{\sigma n}} = \frac{M_\Lambda}{M_n}, \quad r_{\Sigma^- n}^\sigma = \frac{g_{\sigma \Sigma^-}}{g_{\sigma n}} = \frac{M_{\Sigma^-}}{M_n}. \quad (4.14)$$

These values are consistent with those considered appropriate in the nonlinear (σ, ω, ρ) conserving mean-field approximation.

The masses of neutron stars are calculated by using Tollman-Oppenheimer-Volkoff (TOV) equation, energy density and pressure obtained in sec. 3 and sec. 4. They are shown in Fig. 10 as a function of a central energy density, \mathcal{E}_c . The vacuum fluctuation correction softens EOS and reduces the maximum mass of neutron stars about 20 %. It should be noticed that the conserving nonlinear, nonchiral σ, ω mean-field approximation [5] reproduces similar results for K , a_4 , M_{star} , and g_σ , g_ω and nonlinear coefficients, when $m_\sigma = 120.0$ MeV is assumed. Hence, the chiral symmetry breaking mechanism provides a consistent method to understand solutions to nonchiral, nonlinear mean-field models. The result indicates that ρ -meson is necessary to obtain reasonable results for properties of Fermi-liquid and neutron stars. The analysis with chiral $(\sigma, \pi, \omega, \rho)$ model [17] is needed to extract quantitative results.

5 Concluding remarks

In the current extended chiral mean-field model, all the masses of baryons and mesons are produced through chiral symmetry breaking of nonlinear interaction potential, and adjustable free parameters are limited to m_σ , g and g_ω , after hadron masses are identified and fixed in the nuclear domain, *e.g.* $M_N = 939.0$, $m_\pi = 139.0$ and $m_\omega = 783.0$ MeV. The constraints to the chiral mean-field approximation are properties of saturation ($\mathcal{E}/\rho_B - M = -15.75$ MeV, at $k_F = 1.30 \text{ fm}^{-1}$) and the maximum mass of isospin-asymmetric neutron stars ($M_{\text{max}}(n, p, e) \lesssim 2.50 M_\odot$). The mass of σ -meson is determined to

maintain the constraints and given by $m_\sigma \simeq 120$ MeV, which is also necessary so that the interaction potential V is positive and bounded at high densities. The chiral mean-field approximation indicates that a scalar particle close to the mass of π -meson should be needed to produce saturation of nuclear matter.

The effective masses of nucleon and mesons, $M_N^*, m_\sigma^*, m_\omega^*$, are similar to those derived from nonchiral, nonlinear (σ, ω) mean-field approximation. The effective mass of nucleon M_N^*/M_N monotonically decreases, but effective masses of mesons are, $1.0 \lesssim m_\sigma^*/m_\sigma, m_\omega^*/m_\omega$, at or around saturation density. The vacuum fluctuation corrections exhibit repulsive effects for all densities, but after adjusting coupling constants to reproduce properties of saturation and neutron stars, it is examined that the effect of VFC is less significant at saturation than that at high densities. The effect of nonlinear interactions is more important than that of VFC in the Hartree approximation. The similar conclusion is also obtained in the nonchiral, nonlinear (σ, ω, ρ) mean-field approximation. As shown in Fig. 6, ρ -meson gives noticeable contributions, and so, the chiral nonlinear (σ, π, ω) mean-field approximation should be extended and examined by including ρ -meson.

The nonchiral, nonlinear (σ, ω, ρ) mean-field approximations have many adjustable nonlinear coupling constants. The nonlinear coupling constants have upper bound restricted by self-consistent conditions to approximations and properties of saturation and neutron stars [5], which is expected as a manifestation of *naturalness* of nonlinear coefficients [14]. The current chiral mean-field approximation determines all the nonlinear constants in terms of three adjustable parameters: m_σ , g and g_ω . The masses of mesons, m_π and m_ω , are identified and fixed by experimental values, after the chiral symmetry breaking. The nonlinear constants expressed by m_σ , g and g_ω support the properties of naturalness and the bounded values of nonlinear constants given by nonchiral, nonlinear (σ, ω, ρ) mean-field approximation. The self-consistent and optimum solutions to the nonchiral, nonlinear (σ, ω) mean-field approximation with $m_\sigma = 120.0$ MeV become similar to those of the chiral (σ, ω) mean-field approximation, and so, it suggests that chiral symmetry serves to restrict solutions to nonlinear mean-field approximations.

Because the chiral-symmetry breaking relates nonlinear coefficients with hadron masses, the chiral mean-field approximation suggests that nucleon-proton, nucleon-hyperon coupling ratios be given by ratios of hadron masses, such that $r_{\Lambda N}^\sigma = M_\Lambda/M_N \approx r_{\Lambda N}^\omega$ and $r_{\Sigma N}^\sigma = M_\Sigma/M_N \approx r_{\Sigma N}^\omega$. It is remarkable that the values of coupling ratios are consistent with those obtained by the condition at hyperon-onset density, which is determined by the requirement of thermodynamic consistency at saturation of hyperon matter [6]. The coupling ratios produce reasonable density-dependent properties of nuclear matter and neutron stars in the calculation of conserving nonchiral, nonlinear (σ, ω, ρ) mean-field approximation. On the contrary, the coupling ratios given by the SU(6) quark model for vector coupling constants [18, 19] are expected to be $r_{\Lambda N}^\omega = 2/3$ and $r_{\Sigma N}^\omega = 2/3$, but the ratios do not generate consistent results for properties of nuclear and neutron matter. The chiral symmetry breaking is useful to understand relations among nonlinear coupling constants for nonlinear mean-field models.

The effect of VFC is not prominent at saturation density compared with that of nonlinear interactions, but VFC softens EOS at high densities and the EOS would be softened further when hyperons are generated; the fact is also consistent with the result derived from the nonchiral, nonlinear (σ, ω, ρ) mean-field approximation [20]. Since chiral symmetry breaking clarifies relations among nonlinear interactions, it is important to understand how hyperon-onset densities, binding energy and saturation properties of hyperon matter, masses of hadron and hadron-quark stars would be modified by chiral models of hadrons. The chiral symmetry breaking mechanism helps us understand physical meanings of chiral symmetry to masses and coupling constants of hadrons; quantitative analysis in terms of the chiral symmetry breaking may help us understand if σ -meson is a real particle state or a virtual state characteristic to Hartree (mean-field) approximation. The problems of high-energy hadron scatterings and properties of infinite matter such as hadron-quark stars suggest that hadronization from QCD, phase transition from bound state hadrons to quark matter can be one of important topics in the near

future. However, the quantitative analysis in terms of both quantumhadrodynamics (QHD) and QCD is necessary. The current self-consistent chiral mean-field model and other chiral models should be extended to more sophisticated approximations, such as conserving HF and BHF approximations in order to obtain consistent results.

Acknowledgement

We would like to acknowledge the Research Center for Nuclear Physics (RCNP) at Osaka University; especially, Dr. M. Valverde and Dr. J. Hu for their constructive opinions and supports.

References

- [1] B. D. Serot and J. D. Walecka, *Advances in Nuclear Physics*, edited by J. W. Negele and E. Vogt (Plenum, New York, 1986), Vol. 16.
- [2] B. D. Serot and J. D. Walecka, *Int. J. Mod. Phys.*, Vol. E6, (1997) 515.
- [3] B. D. Serot and H. Uechi, *Ann. Phys.* **179** (1989) 272.
- [4] N. K. Glendenning, *Compact Stars*, Springer-Verlag New York, 2000.
- [5] H. Uechi, *Nucl. Phys.* **A780** (2006) 247; H. Uechi, *Nucl. Phys.* **A 799** (2008) 181.
- [6] S. T. Uechi and H. Uechi, *Adv. in High Ene. Phys.*, (2009) 640919; H. Uechi and S. T. Uechi, *The Open Nucl. and Part. Phys. J.*, **V2**, (2009) 47.
- [7] S. T. Uechi and H. Uechi, in preparation (2010).
- [8] B. D. Serot, *Rep. Prog. Phys.* **55** (1992) 1855.
- [9] H. Uechi, *Prog. Theor. Phys.* **111** (2004) 525.
- [10] W. Kohn and L. J. Sham, *Phys. Rev.* **140** (1965) A1133.
- [11] W. Kohn, *Rev. Mod. Phys.*, **71** (1999) 1253.
- [12] H. Uechi, *Prog. Theor. Phys.* **111** (2004) 525.
- [13] J. D. Walecka, *Theoretical Nuclear and Subnuclear Physics*, Oxford University Press, 1995.
- [14] R. J. Furnstahl, B. D. Serot, H-B. Tang, *Nucl. Phys.* **A 598** (1996) 539; R. J. Furnstahl, B. D. Serot, H-B Tang, *Nucl. Phys.* **A 615** (1997) 441.
- [15] W.-Z. Jiang, B.-A. Li, L.-W. Chen, *Phys. Lett.* **B 653** (2007) 184.
- [16] B. D. Serot, *Annals of Phys.*, **322** (2007) 2811.
- [17] B. D. Serot and J. D. Walecka, *Acta Phys. Pol.* **B23** (1992) 655.
- [18] J. Schaffner and I. N. Mishustin, *Phys. Rev.* **C53** (1996) 1416.
- [19] G.-yun Shao, and Y.-xin Liu, *Phys. Rev.* **C79** (2009) 025804.
- [20] Chun T. Uechi and Hiroshi Uechi, arXiv:1003.4815v1 [nucl-th] 25 Mar 2010.
- [21] H. Uechi, *Phys. Rev.* **C41** (1990) 744; *Nucl. Phys.* **A696** (2001) 511